

UT-862

Mass Generation for an Ultralight Axion

Yasunori Nomura¹, T. Watari¹ and T. Yanagida^{1,2}

¹ *Department of Physics, University of Tokyo,
Tokyo 113-0033, Japan*

² *Research Center for the Early Universe, University of Tokyo,
Tokyo 113-0033, Japan*

Abstract

If a global chiral symmetry is explicitly broken by anomalies in nonabelian gauge theories, a pseudo Nambu-Goldstone boson (axion) associated with a spontaneous breakdown of such a global symmetry acquires a mass through nonperturbative instanton effects. We calculate the axion mass assuming a supersymmetric SU(2) gauge theory and show that the axion obtains an extremely small mass when the SU(2) gauge symmetry is broken down at very high energy, say at the Planck scale. We identify the axion with a hypothetical ultralight boson field proposed to account for a small but nonzero cosmological constant suggested from recent cosmological observations.

1 Introduction

A number of recent observations for the total (baryonic and dark) matter density from galaxy clusters suggest that it is significantly less than the critical density [1]. However, most of the inflation models predict that the present universe is spatially flat, namely the total energy density in the universe is equal to the critical one [2]. A nonzero cosmological constant seems to be the simplest candidate to resolve this discrepancy without any contradiction to observations. Furthermore, an introduction of a small cosmological constant provides a natural solution to the so-called “age crisis” of the present universe [3]. It is also very encouraging that the recent studies on the Hubble diagram for type Ia supernovae have supported the presence of the nonzero cosmological constant [4]. Therefore, it is quite reasonable to conclude that totality of the present cosmological observations indicates a small but nonzero cosmological constant.

However, a natural value of the cosmological constant Λ_{cos}^4 is of order the Planck scale, $(M_{\text{pl}} \simeq 2.4 \times 10^{18} \text{ GeV})^4$, or at least of order the supersymmetry (SUSY) breaking scale, $(m_{\text{SUSY}} \simeq 1 \text{ TeV})^4$, in quantum field theories, since anything which contributes to vacuum energy, such as a zero point energy of a field and a vacuum expectation value of a field acts as a cosmological constant. These are many orders of magnitude larger than the value $\Lambda_{\text{cos}}^4 \simeq (3 \times 10^{-3} \text{ eV})^4$ [1, 3, 4] suggested from the cosmological observations. There has been found, so far, no satisfactory solution to this problem [5].

At the present stage of understanding nature, thus, it seems very interesting to assume that the energy density at the true vacuum of the universe is exactly zero, due to some unknown mechanism, and to ask if there is any mechanism that gives a small but nonzero cosmological constant (effective vacuum energy) at the present epoch. This assumption has led to an interesting hypothesis [6] that an ultralight pseudo Nambu-Goldstone boson (axion) field \mathcal{A} dominates the energy density of the present universe and its energy density behaves like a cosmological constant. When the expansion rate of the universe becomes smaller than the \mathcal{A} mass, the axion \mathcal{A} will begin to oscillate around the true vacuum and the universe becomes eventually cold-axion dominated with zero cosmological constant.¹

¹ There has been proposed an alternative mechanism [7] to account for the small cosmological constant.

In Refs. [6, 8], the axion field of mass $m_{\mathcal{A}} \simeq 4 \times 10^{-33}$ eV with a spontaneous breaking scale of a global symmetry $F_{\mathcal{A}} \simeq 2 \times 10^{18}$ GeV has been considered to account for the small cosmological constant $\Lambda_{\text{cos}}^4 \sim (3 \times 10^{-3}$ eV) 4 [1, 3, 4]. However, no convincing mechanism has been proposed to generate such an extremely small mass for the axion.² In this paper, we propose a mechanism to generate the extremely small mass for the axion \mathcal{A} . Instantons of a broken gauge symmetry play a central role on producing the axion mass in our model.

2 A SUSY SU(2) Gauge Model

The model is based on a SUSY SU(2) gauge theory with two doublet quarks Q^i ($i = 1, 2$). Here, we have suppressed the gauge index and i denotes the flavor index. The extension to $SU(N_c)$ gauge theories ($N_c \geq 3$) will be given later. The effective superpotential induced by the SU(2) gauge interaction is [10]

$$W_{\text{dyn}} = \frac{2\Lambda^5}{\epsilon_{ij} Q^i Q^j}, \quad (1)$$

where Λ is the renormalization-group invariant dynamical scale of the SU(2) gauge theory. The Λ is given by

$$\Lambda = M_{\text{pl}} e^{-\frac{2\pi}{b\alpha(M_{\text{pl}})}}. \quad (2)$$

Here, the b is the coefficient of one-loop renormalization-group β function, $b = 5$ in the present model, and the $\alpha(M_{\text{pl}}) \equiv g(M_{\text{pl}})^2/4\pi$ the SU(2) gauge coupling constant at the Planck scale M_{pl} . With this coupling g , the gauge kinetic term \mathcal{L}_{kin} is defined as

$$\mathcal{L}_{\text{kin}} = \int d^2\theta \frac{1}{16g^2} \mathcal{W}^{a\alpha} \mathcal{W}_\alpha^a + \text{h.c.}, \quad (3)$$

where $\mathcal{W}^{a\alpha}$ is the kinetic superfield of the SU(2) vector multiplet ($a = 1 - 3$) and $\alpha = 1, 2$ denotes the spinor index. Notice that the effective superpotential W_{dyn} in Eq. (1) is obtained from nonperturbative instanton effects [10].

² Ref. [8] considers a neutrino mass m_ν as an explicit breaking term of the global symmetry which induces a small mass for the axion. However, if one uses $m_\nu \simeq 5 \times 10^{-2}$ eV suggested from the atmospheric neutrino oscillation [9], one gets too large axion mass.

For the region $Q^i \gg \Lambda$, the running gauge coupling constant $\alpha(Q^i)$ is small and the Kähler potential for Q^i is well described by the minimal form

$$K \simeq Q_i^\dagger Q^i. \quad (4)$$

With this Kähler potential and the dynamically induced superpotential Eq. (1), we easily see a runaway vacuum ($Q^i \rightarrow \infty$) [10]. We introduce a gauge singlet chiral superfield X to stabilize the runaway vacuum in the perturbative regime ($\langle Q^i \rangle \gg \Lambda$). We assume a simple superpotential for X as

$$W_{\text{tree}} = \frac{\kappa}{2} \epsilon_{ij} Q^i Q^j X - V^2 X. \quad (5)$$

Then, the vacuum expectation values for the Q^i and X are

$$\left\langle \frac{1}{2} \epsilon_{ij} Q^i Q^j \right\rangle = \frac{V^2}{\kappa}, \quad (6)$$

$$\langle X \rangle = \frac{\kappa \Lambda^5}{V^4}, \quad (7)$$

where Q^i and X denote boson components of the corresponding superfields. Here, we have assumed that the scale V is much larger than the dynamical scale Λ , *i.e.* $V \gg \Lambda$, otherwise we get $\langle Q^i \rangle \lesssim \Lambda$ where the SU(2) gauge coupling is strong and the perturbative approximation of the Kähler potential in Eq. (4) breaks down.

In this Higgs phase, the SU(2) gauge symmetry is spontaneously broken down and all gauge multiplets become massive absorbing three massless would-be Nambu-Goldstone (NG) multiplets Q^i 's.³ The remaining quark Q^i also has a mass of order $\sqrt{\kappa}V$ together with the X field. Introduction of soft SUSY-breaking masses m_Q^2 and m_X^2 for the scalar Q^i and scalar X fields does not induce any important changes in the mass spectrum and vacuum expectation values in Eqs. (6) and (7) as long as $V \gg m_Q^2, m_X^2$. This is the case even for $m_Q^2, m_X^2 \gg \Lambda$. For instance, the corrections to the vacuum expectation values Eqs. (6) and (7) are of order m_Q^2/V^2 and m_X^2/V^2 , respectively, and the vacuum stays in the weak-coupling regime. This is a crucial point in our analysis, since we are interested

³ In this vacuum, we have an unbroken flavor SU(2)_F symmetry.

in the region $\Lambda^2 \ll m_{\text{SUSY}}^2 \ll V^2 \simeq F_{\mathcal{A}}^2 \simeq M_{\text{pl}}^2$ as we shall see later, where m_{SUSY}^2 denotes the SUSY-breaking mass scale.⁴

We now introduce a pseudo NG chiral superfield $\Phi(x, \theta)$ whose boson components consist of the axion field $\mathcal{A}(x)$ and its scalar partner, saxion $\mathcal{B}(x)$. We consider that the NG superfield is produced by a spontaneous breakdown of some global symmetry at the Planck scale $F_{\mathcal{A}} \simeq 2 \times 10^{18}$ GeV, which is explicitly broken by the SU(2) gauge anomaly.⁵ Then, we expect at low energies that the NG superfield Φ couples to the SU(2) gauge multiplet through the anomaly as

$$\mathcal{L} = \int d^2\theta \frac{1}{128\pi^2} \frac{\Phi}{F_{\mathcal{A}}} \mathcal{W}^{a\alpha} \mathcal{W}_{\alpha}^a + \text{h.c.} \quad (8)$$

Here, the axion $\mathcal{A}(x)$ and saxion $\mathcal{B}(x)$ fields are an imaginary and a real part of the complex boson component of $\Phi(x, \theta)$, respectively.

Together with superpotentials Eqs. (1), (5) and (8), we now obtain a low-energy effective superpotential⁶

$$W_{\text{eff}} = \frac{2\Lambda^5}{\epsilon_{ij} Q^i Q^j} e^{-\Phi/F_{\mathcal{A}}} + \frac{\kappa}{2} \epsilon_{ij} Q^i Q^j X - V^2 X. \quad (9)$$

Then, integration of the X and the Q^i yields

$$W_{\text{eff}} = \frac{\kappa\Lambda^5}{V^2} e^{-\Phi/F_{\mathcal{A}}}. \quad (10)$$

With this superpotential, the \mathcal{B} has a runaway behavior and the effective gauge coupling constant defined by $g_{\text{eff}} = (1/g^2 + \langle \mathcal{B} \rangle / (8\pi^2 F_{\mathcal{A}}))^{-1/2}$ goes to zero. Thus, the \mathcal{A} is massless at the limit $\mathcal{B} \rightarrow \infty$, since the nonperturbative effects which would generate the axion mass vanishes when $g_{\text{eff}} \rightarrow 0$.

⁴ For a region $V^2 \lesssim \Lambda^2 \lesssim m_{\text{SUSY}}^2$, we have $\langle Q^i \rangle \lesssim \Lambda$ and the perturbative description of the Kähler potential Eq. (4) breaks down.

⁵ We neglect nonperturbative effects of gravitational interactions [11] on the axion mass, since we have not yet well understood the quantum gravity.

⁶ We redefine the kinetic term \mathcal{L}_{kin} as $\mathcal{L}_{\text{kin}} = \int d^2\theta (G/4) \mathcal{W}^{a\alpha} \mathcal{W}_{\alpha}^a + \text{h.c.}$ with $G = 1/4g^2 + \Phi/(32\pi^2 F_{\mathcal{A}})$, combining Eq. (3) and Eq. (8). Then, the W_{dyn} is given by $W_{\text{dyn}} = 2M_{\text{pl}}^5 \exp(-32\pi^2 G)/\epsilon_{ij} Q^i Q^j$. This leads to Eq. (9).

The Kähler potential for the Φ superfield should respect the global symmetry $\Phi \rightarrow \Phi + i\delta$. Then, we have the following form of Kähler potential:

$$K = \frac{1}{2}(\Phi + \Phi^\dagger)^2 + \frac{c}{F_A}(\Phi + \Phi^\dagger)^3 + \dots \quad (11)$$

Notice that the linear term has been eliminated by a shift of the Φ field of order F_A , which is accompanied by a slight shift of the gauge coupling constant, g , through Eq. (8). If we consider the SUSY breaking in a hidden sector in supergravity [12], the soft SUSY-breaking mass $m_B^2 \simeq m_{\text{SUSY}}^2$ for the \mathcal{B} field arises with m_{SUSY} being the gravitino mass. The soft mass for the \mathcal{B} field, m_B^2 , stops the runaway behavior of the \mathcal{B} at

$$\langle \mathcal{B} \rangle \simeq \frac{\kappa^3 \Lambda^{10}}{m_B^2 F_A^3 V^6} \left(\frac{V^2}{\kappa} - F_A^2 \right). \quad (12)$$

Since we consider the region $\Lambda^2 \ll m_B^2 \ll F_A^2 \simeq V^2 \simeq M_{\text{pl}}^2$, $\langle \mathcal{B} \rangle \simeq \Lambda^{10}/m_B^2 M_{\text{pl}}^7 \ll F_A$ and we set $\exp(-\mathcal{B}/F_A) \simeq 1$, hereafter. Even then ($g_{\text{eff}} \simeq g \neq 0$), however, the \mathcal{A} field remains massless. This is because there is an anomaly-free global symmetry that is a linear combination of the global symmetry $\Phi \rightarrow \Phi + i\delta$ and an R -symmetry $X \rightarrow \exp(2i\delta)X$, which is broken down by the vacuum expectation value $\langle X \rangle \simeq (\kappa \Lambda^5/V^4)$ in Eq. (7).⁷ Thus, we have to introduce an explicit breaking of the R -symmetry in order to generate an axion mass.

3 Induced Potential for the Axion

We now calculate the axion mass, introducing the R -breaking gaugino mass $m_{\tilde{g}}$ of the SU(2) gauge theory.^{8,9} The effect of the gaugino mass can be incorporated by promoting the gauge coupling to the superfield S as

$$\mathcal{L}_{\text{kin}} = \int d^2\theta \frac{1}{4} S \mathcal{W}^{a\alpha} \mathcal{W}_\alpha^a + \text{h.c.}, \quad (13)$$

⁷ The massless field has a small admixture of the phase of the X field as $\mathcal{A} + (\langle X \rangle / F_A) \varphi_X \simeq \mathcal{A} + (\kappa \Lambda^5 / F_A V^4) \varphi_X$, where $X = \langle X \rangle \exp(i\varphi_X / |\langle X \rangle|)$.

⁸ Introduction of the gaugino mass $m_{\tilde{g}}$ shifts the vacuum expectation value of the \mathcal{B} to $\langle \mathcal{B} \rangle \simeq (\kappa \Lambda^5 / m_B^2 F_A V^2)(32\pi^2 m_{\tilde{g}} / g^2)$ from that in Eq. (12). Even in this case, however, the $\langle \mathcal{B} \rangle$ is still sufficiently small, $\langle \mathcal{B} \rangle \ll F_A$, to fix $\exp(-\mathcal{B}/F_A) \simeq 1$.

⁹ A SUSY-breaking trilinear boson coupling, $(\kappa/2) A \epsilon_{ij} Q^i Q^j X$, may also play a similar role to the gaugino mass $m_{\tilde{g}}$ on inducing the axion mass, where Q^i and X are boson components of the corresponding superfields.

and regarding its F -component as the gaugino mass [13],

$$S = \frac{1}{4g^2} - \frac{i\Theta}{32\pi^2} - \frac{m_{\tilde{g}}}{2g^2}\theta^2. \quad (14)$$

Here, the Θ is the vacuum angle. Then, the dynamical scale Λ defined by Eq. (2) is also promoted to the superfield as $\Lambda^b = M_{\text{pl}}^b \exp(-32\pi^2 S)$. Substituting this into the superpotential Eq. (10) and taking the F -component of the superpotential, we get a mass term for \mathcal{A} as

$$\begin{aligned} \mathcal{L}_{\mathcal{A} \text{ mass}} &\simeq \frac{16\pi^2 m_{\tilde{g}} \kappa \Lambda^5}{g^2 V^2} e^{-i\frac{\mathcal{A}}{F_{\mathcal{A}}}} + \text{h.c.} \\ &= \left| \frac{8\pi m_{\tilde{g}} \kappa \Lambda^5}{\alpha V^2} \right| \cos \left(\frac{\mathcal{A}}{F_{\mathcal{A}}} + \Theta' \right), \end{aligned} \quad (15)$$

where $\Theta' = \Theta + \arg(V^2) - \arg(\kappa) - \arg(m_{\tilde{g}})$. Then, the axion mass is given by

$$m_{\mathcal{A}}^2 \simeq \left| \frac{8\pi \kappa}{\alpha} \frac{m_{\tilde{g}} \Lambda^5}{F_{\mathcal{A}}^2 V^2} \right|, \quad (16)$$

which is extremely small in the region $\Lambda \ll m_{\tilde{g}} \ll F_{\mathcal{A}} \simeq V \simeq M_{\text{pl}}$.¹⁰ This result can be also derived explicitly from the instanton calculation which is given in the Appendix.

It is very intriguing to identify the above axion with the hypothetical ultralight boson field proposed [6, 8] to explain the small cosmological constant $\Lambda_{\text{cos}}^4 \simeq (3 \times 10^{-3} \text{ eV})^4$ suggested from the recent observations. This scenario requires $m_{\mathcal{A}} \simeq 4 \times 10^{-33} \text{ eV}$. For $F_{\mathcal{A}} \simeq V \simeq 2 \times 10^{18} \text{ GeV}$, the extremely small axion mass $m_{\mathcal{A}}$ is obtained with a moderate value of the dynamical scale $\Lambda \simeq 10^{-3} \text{ GeV}$ ¹¹ which corresponds to the SU(2)

¹⁰ Strictly speaking, the axion field is a linear combination of \mathcal{A} and phases of X and Q , since the mass matrix for the axion and the phase fields is given by

$$\mathcal{L} \simeq \frac{1}{2} (\mathcal{A} \varphi_X \varphi_Q) \begin{pmatrix} \frac{8\pi \kappa}{\alpha} \frac{m_{\tilde{g}} \Lambda^5}{F_{\mathcal{A}}^2 V^2} & \kappa^2 \frac{\Lambda^5}{F_{\mathcal{A}} V^2} & \kappa^{7/2} \frac{\Lambda^{10}}{F_{\mathcal{A}} V^7} \\ \kappa^2 \frac{\Lambda^5}{F_{\mathcal{A}} V^2} & \kappa V^2 & \kappa^{5/2} \frac{\Lambda^5}{V^3} \\ \kappa^{7/2} \frac{\Lambda^{10}}{F_{\mathcal{A}} V^7} & \kappa^{5/2} \frac{\Lambda^5}{V^3} & \kappa V^2 \end{pmatrix} \begin{pmatrix} \mathcal{A} \\ \varphi_X \\ \varphi_Q \end{pmatrix}. \quad (17)$$

Here, φ_X and φ_Q are defined as $X = \langle X \rangle \exp(i\varphi_X / |\langle X \rangle|)$ and $\epsilon_{ij} Q^i Q^j / 2 = \langle \epsilon_{ij} Q^i Q^j / 2 \rangle \exp(i\varphi_Q / \sqrt{|\langle \epsilon_{ij} Q^i Q^j / 2 \rangle|})$, respectively. However, the mixing between \mathcal{A} and the phases $\varphi_{X,Q}$ are very small. Thus, the axion is dominantly the \mathcal{A} field and its mass is given by Eq. (16).

¹¹ If the gauge symmetry was not broken, one would need an extremely small dynamical scale $\Lambda \simeq 10^{-3} \text{ eV}$ to obtain such a small axion mass $m_{\mathcal{A}} \simeq 4 \times 10^{-33} \text{ eV}$.

gauge coupling constant $\alpha(M_{\text{pl}}) \simeq 1/38$ at the Planck scale. Here, we have assumed the Yukawa coupling $\kappa \simeq 1$ and the gaugino mass $m_{\tilde{g}} \simeq 1$ TeV. It is very encouraging that it is fairly closed to the standard-model gauge coupling constants at the Planck scale.

We have presented a model that gives naturally an extremely small mass for the ultralight axion through the instanton effects of a broken $SU(2)$ gauge theory.¹² The extension of the above model to the case of $SU(N_c)$ with $N_c - 1$ pairs of quarks and antiquarks is straightforward. In this case, the axion mass is given by

$$m_{\mathcal{A}}^2 \simeq \left| \frac{8\pi\kappa^{N_c-1}}{\alpha} \frac{m_{\tilde{g}}\Lambda^{2N_c+1}}{F_{\mathcal{A}}^2 V^{2N_c-2}} \right|. \quad (18)$$

Even in these models, however, the required value for the gauge coupling constant $\alpha(M_{\text{pl}})$ is the same as in the case of the $SU(2)$.

Acknowledgments

We are grateful for discussions with K.-I. Izawa, M. Kawasaki and K. Kurosawa. Y.N. thanks the Japan Society for the Promotion of Science for financial support. This work was partially supported by “Priority Area: Supersymmetry and Unified Theory of Elementary Particles (# 707)” (T.Y.).

¹² One may use instantons of the electroweak $SU(2)_L$ gauge theory in the SUSY standard model as suggested in Ref. [6]. In this case, the axion acquires a mass from the Kähler potential [14]. Under the presence of $(B + L)$ -violating dimension-five operators, we have estimated the axion mass as $m_{\mathcal{A}}^2 \lesssim |(\mu m_{\text{SUSY}}^3/F_{\mathcal{A}}^2) \exp(-2\pi/\alpha_2(M_{\text{pl}}))| \simeq (10^{-36} \text{ eV})^2$ for $\alpha_2(M_{\text{pl}}) \simeq 1/24$ and $F_{\mathcal{A}} \simeq M_{\text{pl}}$. Here, $\mu \simeq 1$ TeV is the SUSY-invariant Higgs mass. This axion mass is much smaller than the required value. However, if there are matter multiplets which increase the $SU(2)_L$ gauge coupling constant, $\alpha_2(M_{\text{pl}})$, at the Planck scale, one may obtain the desired value for the axion mass.

Appendix

In this Appendix, we calculate the axion potential Eq. (15) in terms of the component-field instanton calculation. The relevant parts of the Euclidean action S_E is given by

$$\begin{aligned} -S_E = & - \int d^4x_E \left(\frac{(\mathcal{B} + i\mathcal{A})}{32\pi^2 F_A} \frac{1}{2} (F_{\mu\nu}^a F_{\mu\nu}^a - F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a) \right. \\ & + \frac{(\mathcal{B} - i\mathcal{A})}{32\pi^2 F_A} \frac{1}{2} (F_{\mu\nu}^a F_{\mu\nu}^a + F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a) \\ & \left. + \mathcal{D}_\mu Q_i^* \mathcal{D}_\mu Q^i - \sqrt{2}i(Q_i^* \lambda^a \frac{\tau^a}{2} \psi^i) - \frac{m_{\tilde{g}}}{2g^2} \lambda^a \lambda^a \right), \end{aligned} \quad (19)$$

where τ^a is the Pauli matrices, λ^a the SU(2) gaugino and $Q^i(x, \theta) = Q^i(x) + \sqrt{2}\theta\psi^i(x)$. We define instanton and anti-instanton configurations as those satisfying $F_{\mu\nu}^a = \tilde{F}_{\mu\nu}^a$ and $F_{\mu\nu}^a = -\tilde{F}_{\mu\nu}^a$, respectively. Then, one anti-instanton effect generates the superpotential Eq. (1) [10].

The zero-mode configuration on one anti-instanton background is given by [15]

$$Q_{li}^*(x) = \frac{(-i\bar{\sigma}_\mu)_{li}(x - x_0)_\mu}{((x - x_0)^2 + \rho^2)^{1/2}} \left(\frac{V^*}{\sqrt{\kappa^*}} \right), \quad (20)$$

$$\psi_\alpha^{li}(x) = \frac{2\sqrt{2}\delta_\alpha^l \rho^2}{((x - x_0)^2 + \rho^2)^{3/2}} \left(\frac{V}{\sqrt{\kappa}} \right) \bar{\zeta}_0^i, \quad (21)$$

$$\lambda_\alpha^a(x) = \frac{-8(\tau^a)_\alpha^\beta (-i\sigma_\mu)_{\beta\dot{\alpha}}(x - x_0)_\mu \rho^2}{((x - x_0)^2 + \rho^2)^2} \bar{\beta}^{\dot{\alpha}} + \frac{-8i(\tau^a)_\alpha^\beta \rho^2}{((x - x_0)^2 + \rho^2)^2} \theta_{0\beta}, \quad (22)$$

where $l, m = 1, 2$ are the SU(2) gauge indices, and $\alpha, \beta = 1, 2$ and $\dot{\alpha} = 1, 2$ denote the left-handed and right-handed spinor indices, respectively. Here, x_0 and ρ represent the center and the size of the instanton, respectively, and $\theta_0, \bar{\beta}, \bar{\zeta}_0$ are anticommuting variables. Substituting this classical configuration into the action Eq. (19) and expanding functional measure around the configuration, we obtain the instanton measure

$$\begin{aligned} d\mu = & d^4x_0 d\rho^2 d^2\theta_0 d^2\bar{\beta} d^2\bar{\zeta}_0 \frac{\kappa}{4\rho^2 V^2} \Lambda^5 \\ & \exp \left(- \int d^4x \left(\frac{(\mathcal{B} + i\mathcal{A})(x)}{F_A} \frac{6\rho^4}{\pi^2 ((x - x_0)^2 + \rho^2)^4} \right) \right. \\ & \left. - 4\pi^2 \rho^2 \left| \frac{V}{\sqrt{\kappa}} \right|^2 + 16i\pi^2 \rho^2 \left| \frac{V}{\sqrt{\kappa}} \right|^2 \bar{\beta} \bar{\zeta}_0 + \frac{m_{\tilde{g}}}{2g^2} 32\pi^2 \theta_0^2 \right). \end{aligned} \quad (23)$$

Here, we have suppressed measures and interactions among fluctuation modes.¹³

Integration of fermionic coordinates, $\theta_0, \bar{\beta}$ and $\bar{\zeta}_0$, yields

$$\begin{aligned} d\mu &= d^4x_0 d\rho^2 16\pi^4 \rho^2 \frac{16\pi^2 m_{\tilde{g}}}{g^2} \frac{V^{*2} \Lambda^5}{\kappa^*} \exp\left(-4\pi^2 \rho^2 \left|\frac{V}{\sqrt{\kappa}}\right|^2\right) \\ &\quad \times \exp\left(-\int d^4x \left(\frac{(\mathcal{B} + i\mathcal{A})(x)}{F_A} \frac{6\rho^4}{\pi^2((x - x_0)^2 + \rho^2)^4}\right)\right). \end{aligned} \quad (24)$$

The factor $m_{\tilde{g}}/2g^2$ comes from an insertion of the gaugino mass, $V^{*2}/\kappa^* = \langle Q^* \rangle^2$ from matter-gaugino vertices, and $\Lambda^5 = M_{\text{pl}}^5 \exp(-2\pi/\alpha(M_{\text{pl}}))$ from an anti-instanton, which can be easily read off from a diagrammatic expression in Fig. 1. Notice that the R -breaking term, $-(m_{\tilde{g}}/2g^2)\lambda^a \lambda^a$, is necessary in order to obtain a nonzero axion potential.

Inspecting Eq. (24), we see that only small-scale instanton contributes to the axion potential, since integration of the instanton size, ρ , is cut off at $\rho \simeq |\sqrt{\kappa}/2\pi V|$ due to the exponential factor $\exp(-4\pi^2 \rho^2 |V/\sqrt{\kappa}|^2)$. Then, the smearing factor $d^4x 6\rho^4/\pi^2((x - x_0)^2 + \rho^2)^4$ is well approximated by the delta function $\delta^4(x - x_0)$ as long as we consider the effective potential at low energies. Then, we can perform the $d\rho^2$ integral and obtain

$$d\mu = d^4x_0 \frac{16\pi^2 m_{\tilde{g}}}{g^2} \frac{\kappa \Lambda^5}{V^2} \exp\left(-\frac{(\mathcal{B} + i\mathcal{A})(x_0)}{F_A}\right). \quad (25)$$

Recalling that the \mathcal{B} is fixed at $\langle \mathcal{B} \rangle \ll F_A$ by the soft SUSY-breaking mass $m_{\mathcal{B}}^2$ (see the text), we determine the axion potential induced by one anti-instanton as

$$\mathcal{L}_{\mathcal{A}, \text{AI}} \simeq \frac{16\pi^2 m_{\tilde{g}}}{g^2} \frac{\kappa \Lambda^5}{V^2} e^{-i \frac{\mathcal{A}}{F_A}}. \quad (26)$$

Similar calculation shows that one instanton induces the axion potential

$$\mathcal{L}_{\mathcal{A}, \text{I}} \simeq \frac{16\pi^2 m_{\tilde{g}}^*}{g^2} \frac{\kappa^* \Lambda^{*5}}{V^{*2}} e^{i \frac{\mathcal{A}}{F_A}}, \quad (27)$$

through the right diagram in Fig. 1. We obtain the axion mass term given in Eq. (15) by summing up both anti-instanton and instanton induced potentials, Eqs. (26) and (27).

¹³ Strictly speaking, ρ is an unstable mode and not a collective coordinate, since the instanton configuration is not a stationary point of the Euclidean action for the spontaneously broken gauge theory.

References

- [1] See, for example, N.A. Bahcall, J.P. Ostriker, S. Perlmutter and P.J. Steinhardt, *Science* **284** (1999) 1481.
- [2] A.H. Guth, *Phys. Rev.* **D23** (1981) 347;
A. Linde, *Particle Physics and Inflationary Cosmology*, (Harwood Academic Publishers, Switzerland, 1990), and references therein.
- [3] L. Krauss and M.S. Turner, *Gen. Rel. Grav.* **27** (1995) 1135;
J.P. Ostriker and P.J. Steinhardt, *Nature* **377** (1995) 600.
- [4] S. Perlmutter *et al.*, *Nature* **391** (1998) 51; *Astrophys. J.* **507** (1998) 46;
A.G. Riess *et al.*, *Astron. J.* **116** (1998) 1009.
- [5] For a review, S. Weinberg, *Rev. Mod. Phys.* **61** (1989) 1.
- [6] M. Fukugita and T. Yanagida, preprint YITP-K-1098 (1994).
- [7] C. Wetterich, *Astron. Astrophys.* **301** (1995) 321;
R.R. Caldwell, R. Dave and P.J. Steinhardt, *Phys. Rev. Lett.* **80** (1998) 1582.
- [8] J.A. Frieman, C.T. Hill, A. Stebbins and I. Waga, *Phys. Rev. Lett.* **75** (1995) 2077;
J.A. Frieman and I. Waga, *Phys. Rev.* **D57** (1998) 4642.
- [9] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, *Phys. Rev. Lett.* **81** (1998) 1562.
- [10] I. Affleck, M. Dine and N. Seiberg, *Phys. Rev. Lett.* **51** (1983) 1026; *Nucl. Phys.* **B241** (1984) 493.
- [11] S. Coleman, *Nucl. Phys.* **B307** (1988) 867.
- [12] For a review, H.P. Nilles, *Phys. Rep.* **110** (1984) 1.
- [13] N. Evans, S.D.H. Hsu and M. Schwetz, *Phys. Lett.* **B355** (1995) 475.
- [14] K. Choi, H.B. Kim and J.E. Kim, *Nucl. Phys.* **B490** (1997) 349;
K. Choi and H. Kim, *Phys. Rev.* **D59** (1999) 072001.
- [15] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.* **B260** (1985) 157.

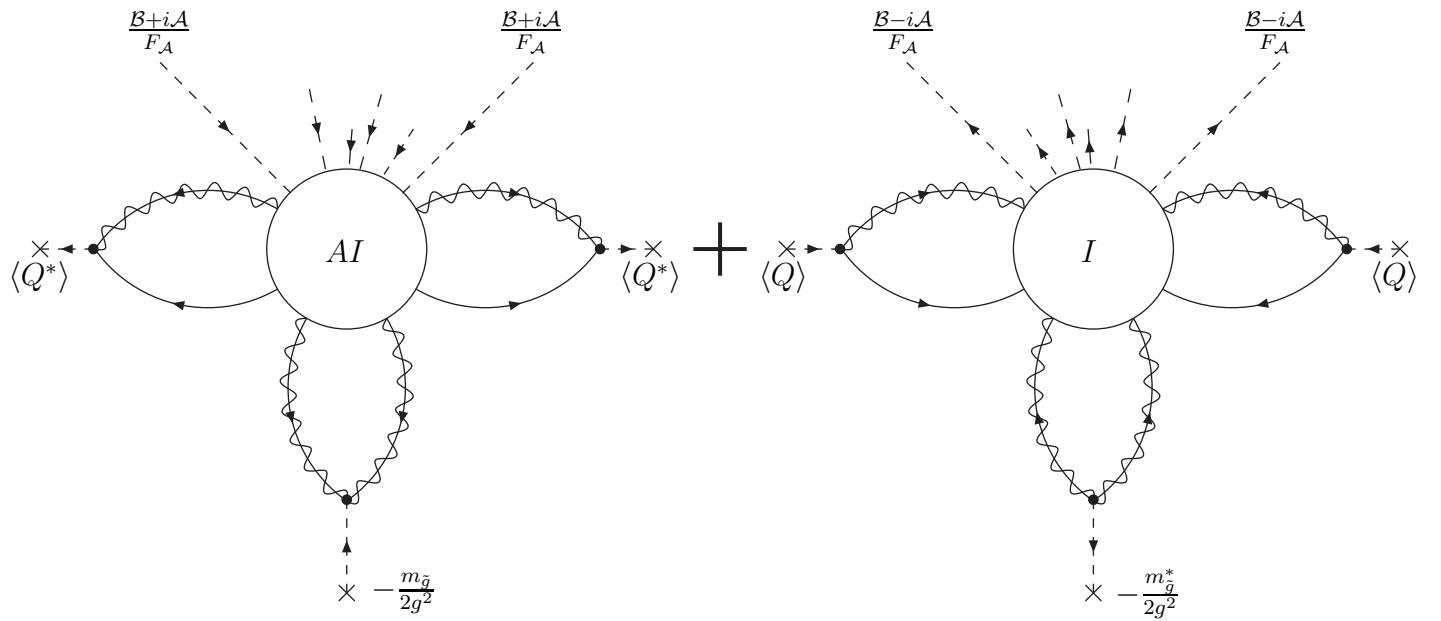


Figure 1: The diagrams which generate the axion potential. The left one represents the anti-instanton contribution giving $\exp(-(\mathcal{B} + i\mathcal{A})/F_A)$ potential, while the right one represents the instanton contribution giving $\exp(-(\mathcal{B} - i\mathcal{A})/F_A)$ potential.